



HANNA ADVANCED EDUCATION SOLUTIONS

COMPUTER SCIENCE SECTION

Fundamental Maths to 3D Vision

Duong-Van Nguyen

duongnguyen.ras@gmail.com

January 11, 2019

History

Author(s)	Date	Version	Description	Approved by
Dr. Duong-V. Nguyen	January 11, 2019	Vers 1.0	Init. version	Dr. Duong-V. Nguyen

Contents

1	Understanding the scope of 3D vision	6
2	Problems Set-up	8
2.1	Mapping environmental structure and its texture	10
2.2	Simultaneous Localization and Map Updating (SLAM)	13
2.2.1	Abstract Formulation	14
2.2.2	Direct-SLAM	17
3	Fundamental Maths	20
3.1	Remarks on Matrix	22
3.1.1	Inverse of a matrix A	23
3.1.2	Null Space	25
3.1.3	Orthogonal and Orthonormal	26
3.2	Eigenvalue and Eigenvector	26
3.3	Singular Value Decomposition (SVD)	29
4	3D Geometry and Camera Model	33
4.1	3D Geometry: Rigid-body Motion	34
4.1.1	Rotation in 3D space	35
4.1.2	Derivation of Rodrigues Rotation Matrix	36
4.1.3	Connection to Lie Group $SO(3)$	38
4.1.4	Lie Group $SE(3)$	40
4.1.5	$Sim(3)$: Similarity Transformation in 3D space	41
4.2	Cameras and Vehicle Coordinate Transformation	44
4.2.1	Camera Mounting	44
4.2.2	Transformation between Coordinates	44
4.3	Mapping from 2D image to 3D camera coordinate system	45
4.3.1	Pinhole-Camera Model	46
4.3.2	Fisheye Camera: Scaramuzza Model	46
4.4	Mapping from 3D camera coordinate system to 2D image	47

4.4.1	Scaramuzza Model for Fisheye Camera	47
4.5	Mapping from 3D camera coordinate system to 2D image	48
4.5.1	Scaramuzza Model for Fisheye Camera	48
4.6	Jacobian with respect to $SE(3)$ transformation group	49
4.6.1	Applied for Scaramuzza Fisheye Camera Model	49
5	Machine Learning for 3D Vision	51

Preface

3D Vision is well known to be a complex problem and only until recently being heavily investigated by the computer vision and robotics societies, typically known as Visual Odometry, Visual SLAM, and Direct SLAM. By naming, this belongs to Perception section inside the overall architect of autonomous systems. However, it will touch most of important knowledge in Computer Vision and Machine Learning. My intention is to write such kind of introduction book incrementally due to my restricted time availability.

Even though there exist many good books about 3D vision and 3D geometry in computer vision or robotics [8] [9] [6] [14] [11], still those require a broad and deep expertise of readers in both mathematics and computer science to follow. The main confrontation that readers usually face is to really connect knowledge from different parts of books to a specific problem that they want to learn how to solve it. Especially, mathematics is supposed to be used to formulate or simplify the problem to be nicer and easier to be solved, nevertheless it turns out to be the most difficult part to be digested from many readers. This is not because of the difficulty in understanding the maths, but the difficulty in understanding the meaning of the maths to which it refer. Meanwhile, in modern Robotics and Computer Vision, much more maths involved in most of problems due to the fact that we expect to have a more accurate solution, a more stable solution, and a solution that can be applied in a real-life application. Consequently, there are great works from Prof. Daniel Cremers on Direct-SLAM [1][4][10][19], which are unfortunately not deeply understood by many researchers. I have observed this through many interviews with Engineers, Ph.D, Post-Doc, etc. and also within discussions in workshops and seminars of conferences. This is WHY I am thinking of writing a book in a bit different way to help readers to more intuitively understanding 3D vision problems and how to solve them.

The difference in the approach of this book compared to a traditional book is that we start from real-life problems, and then trying to formulate it in complex maths, and eventually broken down into simple maths or optimization problems which can be solved mathematically or approximately by machine learning. At a glance, this sounds a bit unusual that we start from something difficult and then deriving it to be simple. However, I believe that this is the correct way for engineers, Ph.D students, and researchers to approach because this is a natural way a complex real-life problem

is solved. Indeed, a real-life problem is usually a very complex problem, we will try to formulate it into a complex but solvable maths - and from then, we will link such formulation with our available knowledge to find a solution. By approaching in this way, Ph.D students, engineers or researchers will in deep know the meaning of maths and why and when they should be used.

Overall, the motivation of this book is to enable Ph.D students and researchers to find a strong connection between Maths and real-life problem solutions, particularly in 3D vision.